FST 3-5 Notes
Topic: The Graph Scale-Change Theorem
GOAL: Apply the Graph Scale Change Theorem to all relations.

SPUR Objectives

C Use the Graph Scale-Change Theorem to find transformation images.

D Describe the effects of translations and scale changes on functions and their graphs.

J Apply the Graph-Translation Theorem or the Graph Scale-Change Theorem to make or identify graphs.

Vocabulary

horizontal and vertical
scale change
scale factor
size change

horizontal scale factor A transformation that maps (x, y) to (ax, y) for all (x, y), where $a \neq 0$ is a constant.

vertical scale change A transformation that maps (x, y) to (x, by), where $b \neq 0$ is a constant.

scale change (in the plane) A transformation that maps (x, y) to (ax, by), where $a \neq 0$ and $b \neq 0$ are constants.

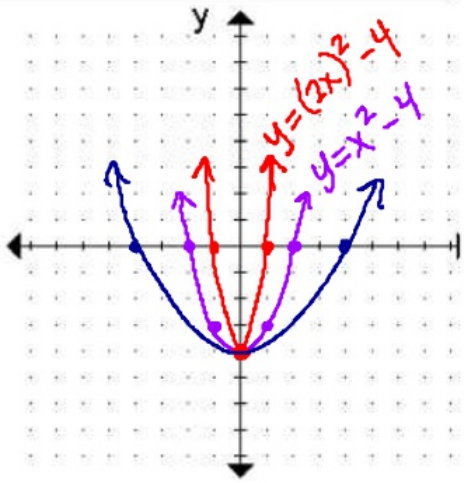
scale factor The nonzero constant by which each data value is multiplied in a scale change.

size change A scale change in which the scale factors are equal; a transformation that maps (x, y) to (kx, ky), where k is a nonzero constant.

$(3x, 3y)$
$(5x, 5y)$

Horizontal Scale Changes

1. Use your calculator to graph $y = x^2 - 4$. down 4
   Sketch the graph in the space provided at the right.
   Be sure to label all intercepts (x and y).
2. Graph $y = (2x)^2 - 4$ on the same set of axes.



Q1: How did the equation change?
x multiplied by 2 then squared

Q2: What changed in your graph? How did it change?
Horizontally shrunk

Q3: What stayed the same in graphs from steps 1 & 2.
Same y-intercept

3. Graph $y = \left(\frac{x}{2}\right)^2 - 4$ on the same set of axes above.

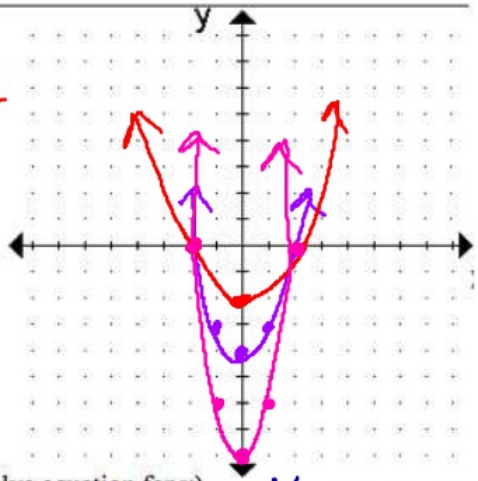
Q4: How did the equation change in step 3 from the original equation in step 1?
x was divided by 2 then squared

Q5: What changed in the graph? How did it change? Horizontally stretched (wider)

Q6: What stayed the same in all 3 graphs?
Same y-intercept

Vertical Scale Changes

4. Graph $y = x^2 - 4$ at the right again. $\frac{2}{2}y = \frac{x^2}{2} - \frac{4}{2}$
   Be sure to label all intercepts (x and y).
5. Graph $2y = x^2 - 4$ on the same set of axes.
   (Hint: Solve equation for y) $y = \frac{x^2}{2} - 2$



Q7: How did the equation change?
x^2 was divided by 2
-4 was divided by 2

Q8: What changed in your graph? How did it change?
Vertical Shrink

Q9: What stayed the same in graphs from steps 4 & 5.
x-intercepts

6. Graph $\frac{y}{2} = x^2 - 4$ on the same set of axes above. (Hint: Solve equation for y) (2) $\frac{y}{2} = 2(x^2 - 4)$

Q10: How did the equation change in step 6 from the original equation in step 4?
It doubled

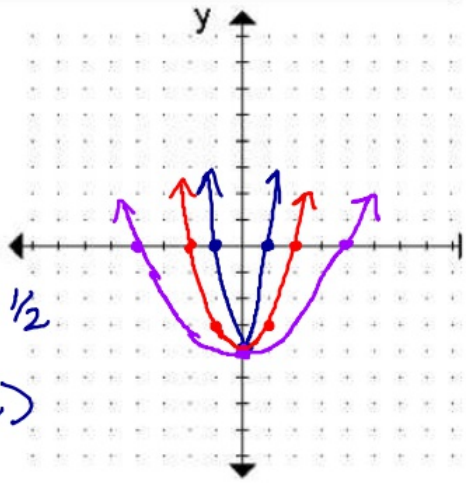
Q11: What changed in the graph? How did it change? Vertical Stretch

Q12: What stayed the same in all 3 graphs?
x-intercepts

$y = 2(x^2 - 4)$
$y = 2x^2 - 8$

Horizontal Scale Changes

1. Use your calculator to graph $y = x^2 - 4$.
   Sketch the graph in the space provided at the right.
   Be sure to label all intercepts (x and y).
2. Graph $y = (2x)^2 - 4$ on the same set of axes.



Q1: How did the equation change?
x was multiplied by 2 then squared

Q2: What changed in your graph? How did it change?
Horizontally shrunk by a factor of ½ (½x, y)

Q3: What stayed the same in graphs from steps 1 & 2.
same vertex (same y-intercept)

3. Graph $y = \left(\frac{x}{2}\right)^2 - 4$ on the same set of axes above.

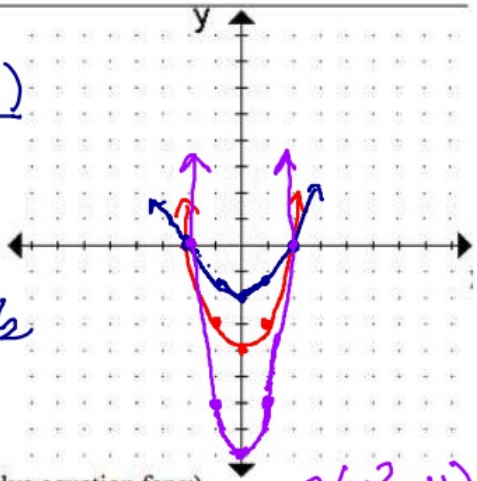
Q4: How did the equation change in step 3 from the original equation in step 1?
x was divided by 2 Then squared

Q5: What changed in the graph? How did it change? Horizontal stretch by a factor of 2 (2x, y)

Q6: What stayed the same in all 3 graphs?
The vertex was the same (same y-intercept)

Vertical Scale Changes

4. Graph $y = x^2 - 4$ at the right again.
   Be sure to label all intercepts (x and y).
5. Graph $\frac{2y}{2} = \frac{x^2}{2} - \frac{4}{2}$ on the same set of axes. $y = \frac{(x^2-4)}{2}$
   (Hint: Solve equation for y) $y = \frac{x^2}{2} - 2$



Q7: How did the equation change?
x^2 was divided by 2 and -4 was divided by 2

Q8: What changed in your graph? How did it change?
Vertically shrunk by a factor of ½ (x, ½y)

Q9: What stayed the same in graphs from steps 4 & 5.
x-intercepts

6. Graph $\frac{y}{2} = x^2 - 4$ on the same set of axes above. (Hint: Solve equation for y) $y = 2(x^2-4)$ $y = 2x^2 - 8$

Q10: How did the equation change in step 6 from the original equation in step 4?
It was doubled

Q11: What changed in the graph? How did it change? Vertex moved down Vertical stretch by a factor of 2 (x, 2y)

Q12: What stayed the same in all 3 graphs?
x-intercepts

Graph Scale-Change Rule: S(x, y) = (ax, by)

Where: a is the horizontal scale factor
b is the vertical scale factor

Recall

Translation Rule: $T(x,y) \rightarrow (x+h, y+k)$

In equation form, the 'opposite' happened – addition in the translation rule corresponded to subtraction in the equation

If $y = f(x)$ was translated by the rule above, the new equation would be
$y - k = f(x - h)$

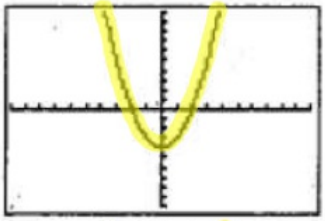
The same 'opposite' happens between the rule for scale change and the equation

Multiplication in rule corresponds to division in equation

Division in rule corresponds to multiplication in equation

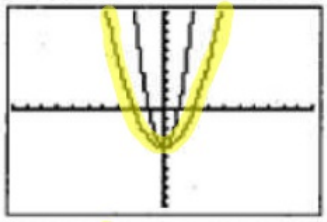
From Activity:

$y = x^2 - 4$



original equation

$y = (2x)^2 - 4$

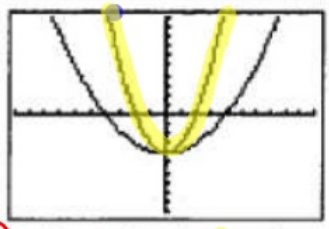


horizontal shrink

$S(x,y) \rightarrow (\frac{x}{2}, y)$

$(\frac{1}{2}x, y)$

$y = \left(\frac{x}{2}\right)^2 - 4$

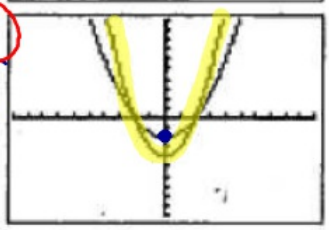


horizontal stretch

$S(x,y) \rightarrow (2x, y)$

$\frac{2y}{2} = \frac{x^2}{2} - \frac{4}{2}$

$y = \frac{x^2}{2} - 2$



Vertical shrink

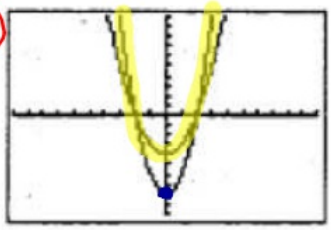
$S(x,y) \rightarrow (x, \frac{y}{2})$

$(x, \frac{1}{2}y)$

$\frac{y}{2} = x^2 - 4$

$y = 2(x^2 - 4)$

$y = 2x^2 - 8$



Vertical stretch

$S(x,y) \rightarrow (x, 2y)$

horizontal scale change (ax, y) when $a>1$ horizontal stretch; when $a<1$ horizontal shrink

Vertical scalechange (x, by) when $b>1$ vertical stretch; when $b<1$ vertical shrink

Additional Example 1

Sketch and compare the graphs of $y = |x|$ and $\frac{y}{4} = |6x|$. Describe the transformation that maps the first graph onto the second.

equation

$y = 4|6x|$

* multiplication in rule corresponds to division in equation
* Division in rule corresponds to multiplication in equation

$$S(x,y) \rightarrow \left(\frac{x}{6}, 4y\right) \rightarrow \left(\frac{1}{6}x, 4y\right)$$

horizontal shrink by $1/6$

vertical stretch by 4

Additional Example 2

The line $41x - 29y = 700$ contains the points $(39, 31)$ and $(10, -10)$. Use this information to obtain two points on the line with equation $20.5x - 87y = 700$.